

Semileptonic width ratios among beauty hadrons

I.I. Bigi,^a Th. Mannel,^b N. Uraltsev^{a,b,c}

^a *Department of Physics, University of Notre Dame du Lac, Notre Dame, IN 46556, USA*

^b *Theoretische Physik 1, Fachbereich Physik, Universität Siegen
D-57068 Siegen, Germany*

^c *St. Petersburg Nuclear Physics Institute, Gatchina, St. Petersburg 188300, Russia*

Abstract

We present predictions based on the heavy quark expansion in QCD. We find $SU(3)$ breaking in B mesons suppressed in the framework of the HQE. B_s is expected to have the semileptonic width about 1% lower and Λ_b about 3% higher when compared to $\Gamma_{sl}(B_d)$. The largest partial-rate preasymptotic effect is Pauli interference in the $b \rightarrow u \ell \nu$ channel in Λ_b , about +10%. We point out that the Ω_b semileptonic width is expected not to exceed that of B_d and may turn out to be the smallest among stable b hadrons despite the large mass. The underlying differences with phase-space models are briefly addressed through the heavy mass expansion.

Contents

1	Introduction	2
2	Heavy quark expansion with $SU(3)$ breaking	3
2.1	μ_G^2	5
2.2	μ_π^2	5
2.3	ρ_D^3	6
2.4	Higher orders in $1/m_b$	7
2.5	$SU(3)$ breaking and heavy quark sum rules	8
3	The heavy quark expansion for $\Gamma_{\text{sl}}(\Lambda_b)$	9
3.1	μ_π^2	9
3.2	ρ_D^3 and higher orders	9
3.3	A note on $\Gamma_{\text{sl}}(\Omega_b)$	10
4	Charmless semileptonic decays	11
5	The semileptonic widths in the phase-space model	12
5.1	On the $\bar{\Lambda}$ effects	14
6	Conclusions	15

1 Introduction

At the LHC a new dedicated flavor experiment has come into operation, namely the LHCb. Big data sets of B_u and B_d transitions have been investigated at the B factories by Belle and BaBar and at FNAL by CDF and D0; smaller data sets for B_s and Λ_b decays have been studied by CDF and D0 (and also by Belle for B_s). LHCb will generate even much larger sets of weakly decaying beauty mesons and baryons, including Ξ_b and Ω_b . The lifetimes of Λ_b , Ξ_b and Ω_b will be well measured as will be the B_s total width and $\Delta\Gamma_{B_s}$. LHCb will also analyze semileptonic channels. The future Super-Flavor Factories will measure inclusive semileptonic rates for these b hadrons with good accuracy. The anticipated experimental precisions should be matched with reliable theoretical predictions incorporating nonperturbative effects.

In a recent paper [1] a question was raised about the difference between the inclusive semileptonic decay rates of different heavy flavor hadrons; beauty particles represent the most interesting case in this respect. It is appropriate to summarize the up-to-date predictions of the existing QCD-based theory. In retrospect, the numerical aspects of the predictions for the differences in the semileptonic widths derived from the Heavy Quark Expansion (HQE) have been addressed so far occasionally [2, 3], with more emphasis on the related theoretical aspects or as a supplementary tool to other studies. The main

attention has been dedicated to the lifetimes of beauty particles, even though it had been appreciated from the earliest days of the HQE that the semileptonic decays generally allow more precise predictions.

We find the HQE predictions for the inclusive beauty semileptonic decay rates differ significantly from those in Ref. [1] which are mostly based on a simple phase-space model. An experimental verification would serve as yet another instructive example of the role of the consistent treatment of the nonperturbative strong dynamics. We also briefly consider the major differences with the naive models in the context of the heavy mass expansion.

2 Heavy quark expansion with $SU(3)$ breaking

The treatment of the nonperturbative QCD effects on the fully inclusive decay rates of the heavy flavor hadrons is provided by the OPE-based HQE [4, 5].¹ Its two principal points in the present context are:

- The overall decay probability does not have corrections linear in $1/m_Q$. In other words, the mass itself of the decaying hadron does not affect the decay rate; the rate is rather determined by the (short-distance) masses of the quarks appearing in the weak decay Lagrangian. Hadron masses in the final state are only indirectly related to the width through various sum rules constraining the relevant nonperturbative QCD expectation values in the decaying heavy flavor hadron.
- The tower of the local heavy quark operators and their coefficients for a given quark-level channel appearing in the $1/m_Q$ -expansion of the widths are universal. This means in turn, that all the dependence on a decaying hadron lies in the expectation values of these universal series of operators. This feature was not self-manifest for the historically first considered nonperturbative effects, namely weak annihilation (WA) and Pauli interference (PI) [7], in particular when introduced through simple quark diagrams. It was treated properly later [8, 2].

As a consequence, the analysis of the $SU(3)$ -breaking pattern in $\Gamma_{\text{sl}}(B_s)$ vs. $\Gamma_{\text{sl}}(B_d)$ requires the estimates of the difference in the expectation values of the leading heavy quark operators between B_s and B in the series [9]

$$\Gamma_{\text{sl}}(B) = \frac{G_F^2}{192\pi^3} |V_{cb}|^2 m_b^5 z_0 \left(\frac{m_s^2}{m_b^2} \right) \left[1 + c_\pi \frac{1}{2m_b^2} \frac{\langle B | \bar{b} (i\vec{D})^2 b | B \rangle}{2M_B} + c_G \frac{1}{2m_b^2} \frac{\langle B | \bar{b} (\frac{i}{2} \sigma_{\mu\nu} G^{\mu\nu} b | B \rangle}{2M_B} + \right. \\ \left. c_D \frac{1}{m_b^3} \frac{\langle B | \bar{b} (-\frac{1}{2} \vec{D} \vec{E}) b | B \rangle}{2M_B} + \frac{32\pi^2}{z_0 m_b^3} \frac{\langle B | \bar{b} \vec{\gamma} (1 - \gamma_5) c \bar{c} \vec{\gamma} (1 - \gamma_5) b | B \rangle_{\text{IC}}}{2M_B} + \dots \right], \quad (1)$$

where the ellipses stand for the higher orders in $1/m_b$.

¹The idea that the total weak decay rates of heavy flavor hadrons asymptotically approach the free-quark rate goes back to the paper [6].

The last term explicitly given in Eq. (1) describes the subset of the higher-order non-perturbative corrections generically referred to as ‘Intrinsic charm’ (IC) – a term not fully adequate here from a theoretical perspective. Various corrections to the Wilson coefficients – including the often technically challenging perturbative renormalization important for precision evaluation of the inclusive widths – are not mandatory here. Nor is generally a scrupulous treatment of the renormalization point in the heavy quark operators, as long as it is of a reasonable scale. Likewise, accounting for the significant τ -lepton mass is not critical unless a precise prediction specifically for the channel $b \rightarrow c(u) \tau \nu_\tau$ [10] is aimed at. For the decay width mediated by the $b \rightarrow u \ell \nu$ transitions the four-fermion ‘WA’ operator $\bar{b}u \bar{u}b$ is required replacing the ‘IC’ expectation value, which is not a $SU(3)$ or V -spin singlet.

A summary of the breakdown of the estimated power corrections in the semileptonic B decays can give a starting idea about the expected effects [11]:

$$\begin{aligned}
\frac{1}{m_b^2} : \quad & \frac{\delta \mu_\pi^2 \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx -1\%, \quad \frac{\delta \mu_G^2 \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx -3.5\% \\
\frac{1}{m_b^3} : \quad & \frac{\delta \rho_D^3 \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx -3\% \\
\frac{1}{m_b^4} : \quad & \frac{\delta \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx 0.5\% \\
\frac{1}{m_Q^5} : \quad & \frac{\delta_{\text{IC}} \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx 0.7\%, \quad \frac{\delta_{\text{tot}} \Gamma_{\text{sl}}}{\Gamma_{\text{sl}}} \approx 0.5\%; \quad (2)
\end{aligned}$$

these estimates were derived for non-strange B mesons. The spin-orbit expectation value ρ_{LS}^3 emerging at the $1/m_b^3$ level enters the width only as a part of the full Lorentz-invariant quantity μ_G^2 ; its effect in the differential distributions is typically insignificant.

An important feature is the smallish impact of the kinetic operator. Its coefficient $c_\pi = -1$ is universally suppressed in the integrated rates [12]; in a certain sense it can even be regarded as vanishing. Therefore a realistic variation of μ_π^2 will not lead to a relevant direct effect on $\Gamma_{\text{sl}}(B_s)/\Gamma_{\text{sl}}(B_d)$. We thus see that the main effect potentially comes from the chromomagnetic interaction or from the Darwin term. Assuming the typical scale of about 30% for the $SU(3)$ violation in the expectation values we may a priori expect a difference of around two percent in $\Gamma_{\text{sl}}(B_s)$. Our actual prediction turns out lower and definitely favors the suppression of the B_s width.

We present a more detailed discussion of the $SU(3)$ breaking effects in the following. The primary inputs on the experimental side are the masses of B , B^* and D , D^* , both strange and non-strange. To build a consistent physical picture of the effects we often confront our expectations qualitatively to the explicit pattern of the heavy quark expansion numerically studied in Ref. [13] in the context of the exactly solvable ‘t Hooft model [14], a large- N_c limit of QCD in two dimensions.

2.1 μ_G^2

The chromomagnetic expectation value μ_G^2 is most directly extracted from the hyperfine splitting $M_{B^*} - M_B$:

$$M_{B^*} - M_B \simeq \frac{4}{3} \tilde{c}_G \frac{\mu_G^2}{2m_b}, \quad \tilde{c}_G \approx 1. \quad (3)$$

Using $M_{B^*} - M_B = 45.78 \pm 0.35$ MeV, $M_{B_s^*} - M_{B_s} = 49.0 \pm 1.5$ MeV we arrive at a tiny $SU(3)$ violation in the hyperfine splitting:

$$\frac{\mu_G^2(B_s)}{\mu_G^2(B_d)} \simeq 1.07 \pm 0.03; \quad (4)$$

qualitatively it can be regarded as nearly absent. In principle, the $1/m_b$ corrections to the relation (3) are about 15% [15]. However, they are smaller in the ratio; this is supported by the similar equality in the D system:

$$M_{D_s^*} - M_{D_s} = 143.8 \pm 0.4 \text{ MeV} \quad \text{vs.} \quad M_{D^{*+}} - M_{D^+} = 140.65 \pm 0.1 \text{ MeV}. \quad (5)$$

Taken at face value the $SU(3)$ breaking (4) would lead to a small shift

$$\delta_{\mu_G^2} \frac{\Gamma_{\text{sl}}(B_s)}{\Gamma_{\text{sl}}(B_d)} \simeq -0.25\%. \quad (6)$$

The actual uncertainty is probably at least two thirds of the value itself.

2.2 μ_π^2

The overall impact of the kinetic operator is a few times smaller than the chromomagnetic effect; therefore no high precision in it is required. Its $SU(3)$ -nonsinglet component can be estimated comparing the observed mass shifts in B and D systems [4]; one can either use the spin-averaged combinations or just the pseudoscalar masses proper, since the hyperfine splitting turns out nearly $SU(3)$ -blind. We have

$$\frac{3M_{B_s^*} + M_{B_s}}{4} - \frac{3M_{B^*} + M_B}{4} = (86.8 + 2.4) \text{ MeV} \quad \text{vs.} \quad \frac{3M_{D_s^*} + M_{D_s}}{4} - \frac{3M_{D^{*+}} + M_{D^+}}{4} = (98.9 + 2.6) \text{ MeV}; \quad (7)$$

$$M_{B_s} - M_B = 86.8 \text{ MeV} \quad \text{vs.} \quad M_{D_s} - M_D = (98.9 \pm 0.3) \text{ MeV}. \quad (8)$$

The differences amount to $\bar{\Lambda}_s - \bar{\Lambda}$, which is therefore about 85 MeV. The variation in the estimate between beauty and charm is an effect of $1/m_Q$ (and higher) terms. Neglecting the higher-order corrections and using $m_b = 4.6$ GeV, $m_c = 1.25$ GeV we get

$$\mu_\pi^2(B_s) - \mu_\pi^2(B_d) \simeq 0.041 \text{ GeV}^2 \quad (9)$$

in either way. (Certain allowance should be left for the electromagnetic effects.) This means a 10% $SU(3)$ breaking in μ_π^2 and only a per mil stronger suppression of $\Gamma_{\text{sl}}(B_s)$ vs. $\Gamma_{\text{sl}}(B_d)$.

One should, however, consider that the $1/m_Q^2$ corrections in the meson masses are as large as 30 to 50 MeV in charm [16]. Even though their $SU(3)$ symmetry softens the impact on the splitting in question, the quality of the $SU(3)$ relations generally deteriorates for higher-order expectation values. To account for this we shall adopt an interval of larger $SU(3)$ breaking; our expectations actually center about the twice larger effect than in Eq. (9):

$$\mu_\pi^2(B_s) - \mu_\pi^2(B_d) \simeq 0.08 \text{ to } 0.1 \text{ GeV}^2 ; \quad (10)$$

i.e. up to 25%. This does not really affect directly $\Gamma_{\text{sl}}(B_s)$ at an appreciable level; it refers more to gaining a self-consistent dynamic picture of the heavy mesons.

2.3 ρ_D^3

The Darwin expectation value ρ_D^3 from dimension-six operators for B mesons is a priori less certain – as expected when the dimension of the operators is increased. A reasonable estimate for it is provided by the exact sum rules applicable in the heavy quark limit. Assuming their dominance by the lowest P -wave states with $\bar{\epsilon}_P \simeq 400$ to 450 MeV we obtain

$$\rho_D^3 \simeq \bar{\epsilon}_P \mu_\pi^2 \simeq 0.18 \text{ GeV}^3, \quad \rho_D^3 \gtrsim \frac{2}{3} \frac{(\mu_\pi^2)^2}{\bar{\Lambda}} \simeq 0.18 \text{ GeV}^3 \quad (11)$$

apparently well fitting the data. Blindly using the same relation in B_d and in B_s for comparing ρ_D^3 and $\rho_D^3(B_s)$ we would have gotten

$$\frac{\rho_D^3(B_s)}{\rho_D^3} \simeq \left(\frac{\mu_\pi^2(B_s)}{\mu_\pi^2} \right)^2 \frac{\bar{\Lambda}}{\bar{\Lambda}_s} \quad \text{with} \quad \bar{\Lambda}_s - \bar{\Lambda} \simeq 82 \text{ MeV}, \quad (12)$$

and thus a ratio about 1.27 assuming a 20% increase in $\mu_\pi^2(B_s)$ compared to $\mu_\pi^2(B_d)$, see Eq. (10). (This ratio becomes 1.07 if using the number from Eq. (9).) However, the validity of the estimates like Eq. (11) may not necessarily be equal for different spectator mass in B meson; in other words, even a fair overall accuracy of such relations does not automatically apply to their derivative in respect to spectator mass m_s as would be required to justify Eq. (12).²

An alternative evaluation of ρ_D^3 dating back to 1993 [17] relies on the vacuum factorization estimate for the four-fermion heavy quark operator the Darwin operator reduces to by the gauge field equations of motion:

$$\rho_D^3 \approx \frac{g_s^2}{18} f_B^2 M_B. \quad (13)$$

Since the factorized contribution is leading in N_c for the Darwin operator, it is a reasonable guess.³ For the ratio the precise scale entering the strong coupling does not matter, and

²We did observe this to hold in the 't Hooft model when studied the effects of duality violation in its setting.

³The situation is different in this respect for the typical four-quark operators encountered in the analysis of the flavor-dependent corrections in the lifetimes.

we get

$$\frac{\rho_D^3(B_s)}{\rho_D^3} \approx \frac{f_{B_s}^2}{f_B^2}. \quad (14)$$

Such a relation is exact in the heavy quark limit in the 't Hooft model [18, 19].

The actual values of the axial decay constants in B mesons are not yet well known, although the question has gotten much attention in the past years. It was suggested in the 1980s [20] that

$$\frac{f_{B_s}}{f_B} \gtrsim \frac{f_K}{f_\pi}, \quad (15)$$

this leads to $f_{B_s}^2/f_B^2 \approx 1.4 \div 1.7$, a ratio preferred nowadays. Regardless of a concrete reasoning, it is more than just plausible that f_B – a wavefunction density at origin – increases with the mass of the light quark in the meson for both heavy-light and light-light bound states.

Various arguments lead us to suggest that $\rho_D^3(B_s)/\rho_D^3$ is larger than unity; the question is rather by how much $SU(3)$ is violated in this parameter. We think that a priori the natural scale is 40 to 50%. On the other hand, the heavy quark relations seem to favor somewhat softer $SU(3)$ breaking effects around 20%. Since both lines of reasoning are quite general although more qualitative, the most natural scenario seems to lie in between:

$$\frac{\rho_D^3(B_s)}{\rho_D^3} \approx 1.25. \quad (16)$$

With the significant negative coefficient $c_D \approx -16$ [21] this yields an additional suppression of $\Gamma_{\text{sl}}(B_s)$ by about 0.8%.

2.4 Higher orders in $1/m_b$

The number of operators in the series for $\Gamma_{\text{sl}}(B)$ proliferates fast in higher orders. Moreover, the $SU(3)$ symmetry in their individual expectation values can be strongly violated being restored to the ‘normal’ level only in the aggregate effect on the observables. Therefore, a detailed analysis paralleling the one given above would become less and less meaningful. A more reasonable perspective is to calibrate the overall effect and to assume an up to 50% breaking of $SU(3)$ in higher orders. Furthermore, we would generally expect their enhancement in B_s vs. B_d ; this is justified provided the net effect does not come as a result of significant cancellations.

The recent analyses estimated the effects of higher-order OPE terms at the one percent level being dominated by the so called IC corrections [22, 23, 11]:

$$\frac{\delta_{1/m_b^4} \Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} \approx 0.5\%, \quad \frac{\delta_{\text{IC}} \Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} \approx 0.7\%, \quad \frac{\delta_{1/m_b^5}^{\text{tot}} \Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} \approx 0.5\%. \quad (17)$$

Based on the above prescription we expect

$$\frac{\delta^{\text{hi ord}} \Gamma_{\text{sl}}(B_s) - \delta^{\text{hi ord}} \Gamma_{\text{sl}}(B)}{\Gamma_{\text{sl}}(B)} \lesssim 0.5\%. \quad (18)$$

To summarize: We expect the dominant $SU(3)$ -breaking correction to $\Gamma_{sl}(B_s)/\Gamma_{sl}(B_d)$ to come from the Darwin operator constituting an impact of about -0.8% , followed by the smaller negative effects from the chromomagnetic and kinetic operators with -0.25% and -0.15% , respectively; the latter two could be offset by a small relative enhancement of the B_s width by about 0.5% from higher-order power corrections.

2.5 $SU(3)$ breaking and heavy quark sum rules

It is instructive to recall that the leading heavy quark nonperturbative parameters are given by the moments of the positive SV structure functions of the respective B mesons:

$$\varrho^2 - \frac{1}{4} = \int W_+(\varepsilon) d\varepsilon, \quad \overline{\Lambda} = 2 \int W_+(\varepsilon) \varepsilon d\varepsilon, \quad \mu_\pi^2 = 3 \int W_+(\varepsilon) \varepsilon^2 d\varepsilon, \quad \rho_D^3 = 3 \int W_+(\varepsilon) \varepsilon^3 d\varepsilon. \quad (19)$$

For the $SU(3)$ breaking the difference $W_+^{(B_s)} - W_+^{(B_d)}$ enters. Of course, the moment relations are less constraining when the function is no longer positive. Nevertheless certain qualitative conclusions can be drawn.

We have observed some increase in the relative $SU(3)$ breaking for higher moments of $W_+(\varepsilon)$ – from the first ($\overline{\Lambda}$) to the third (ρ_D^3), which looks natural. Extrapolating this to the zeroth moment we would arrive to expect a small increase in the IW slope in B_s compared to B_d .

Let us note that the particular $SU(3)$ -breaking pattern

$$\overline{\Lambda}_s - \overline{\Lambda} = 82 \text{ MeV}, \quad \mu_\pi^2(B_s) - \mu_\pi^2(B_d) = 0.06 \text{ GeV}^2, \quad \rho_D^3(B_s) - \rho_D^3(B_d) = 0.04 \text{ GeV}^3 \quad (20)$$

would be well described, for $\overline{\Lambda}$ and μ_π^2 , by a small increase in $2|\tau_{3/2}|^2 + |\tau_{1/2}|^2$ by 0.1 at $\varepsilon \simeq 0.45 \text{ GeV}$; yet it would be somewhat worse for ρ_D^3 where additional contributions from higher strange P -waves are then preferred. This qualitative solution (not unique, of course) would imply the increase in the strange IW slope by about 0.1:

$$\varrho_{B_s}^2 - \varrho^2 \approx 0.1. \quad (21)$$

In the 't Hooft model the overall m_s -dependence has been observed to be weaker both in the leading heavy quark parameters and in the meson axial constant; the dominance of the first P -wave excitation in the sum rules was excellent for either spectator mass. If we accept the same to be the case in actual QCD we would end up with a lower $SU(3)$ breaking in the Darwin value and in the f_{B_s}/f_B ratio than it has been estimated in the previous sections.

We have found a further reduced $SU(3)$ breaking in μ_G^2 , which is smaller than can be expected from any sort of nonrelativistic quark model. This, however is more natural for a highly relativistic bound state by virtue of the sum rules: the exact relation equating the zeroth moment of the $W_-(\varepsilon)$ SV structure function to the spin of the light cloud [24] mandates complete absence of the m_s effects in this moment, a property apparently reflected, in a weaker form, in the second moment yielding μ_G^2 .

Let us note in passing that in the BPS limit [25] $\mu_\pi^2 = \mu_G^2$ for non-strange B mesons the first-order $SU(3)$ -breaking corrections in the difference $\mu_\pi^2 - \mu_G^2$ (as well as in ϱ^2) must vanish starting with terms $\propto m_s^2$. The proximity to the BPS regime may therefore explain the apparent smallness of the relative $SU(3)$ -breaking in the lowest-dimension nonperturbative heavy quark parameters.

3 The heavy quark expansion for $\Gamma_{\text{sl}}(\Lambda_b)$

No unitary symmetry per se relates the decay rates of beauty baryons and mesons; the corresponding expectation values must be analyzed separately.

Since the light degrees of freedom in Λ_b carry no spin in the heavy quark limit, the structure of the $1/m_Q$ series simplify compared to B mesons: only spin-singlet expectation values survive. In particular, the contribution from the chromomagnetic operator vanishes to the leading order in $1/m_b$.

3.1 μ_π^2

As suggested in Ref. [4], we have estimated $\mu_\pi^2(\Lambda_b) - \mu_\pi^2$ following the analogue of Eq. (8) via

$$\begin{aligned} \bar{\Lambda}_\Lambda - \bar{\Lambda} &\simeq M_{\Lambda_b} - \bar{M}_B \approx 305 \text{ MeV} \\ \left(\frac{1}{2m_c} - \frac{1}{2m_c}\right) [\mu_\pi^2(\Lambda_b) - \mu_\pi^2] + \mathcal{O}\left(\frac{1}{m_Q^2}\right) &\simeq (M_{\Lambda_c} - \bar{M}_D) - (M_{\Lambda_b} - \bar{M}_B) \simeq 5.4 \text{ MeV} \end{aligned} \quad (22)$$

which literally would yield $\mu_\pi^2(\Lambda_b) - \mu_\pi^2 \simeq 0.02 \text{ GeV}^2$. However, higher-order corrections in $1/m_c$ are more significant. In particular, the $1/m_c^2$ terms were estimated [16] to be about -45 MeV in \bar{M}_D , which by itself would raise the μ_π^2 difference by 0.15 GeV^2 . Little is known at the moment about them in Λ_c ; they can generally be expected similar in size. We thus estimate

$$\mu_\pi^2(\Lambda_b) - \mu_\pi^2 \approx 0.1 \pm 0.1 \text{ GeV}^2; \quad (23)$$

the uncertainty figure here is not iron-clad, yet the precise value is not of a direct importance for the width difference. Therefore we conclude that

$$\delta_{\mu_\pi^2} \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B_d)} \approx -0.25\%. \quad (24)$$

3.2 ρ_D^3 and higher orders

Little as well is known directly about the Darwin expectation value in Λ_b . Its value was estimated in Ref. [26] to be about 0.15 GeV^3 – somewhat smaller than ρ_D^3 in B mesons. The related estimate of the kinetic expectation value in Λ_b yielded the result quite close to Eq. (23). Our expectation then is that

$$\delta_{\rho_D^3} \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B_d)} \simeq (1 \pm 1.5)\%, \quad (25)$$

tentatively the next-to-largest effect after the vanishing of the chromomagnetic term.

The question of the higher-order corrections is somewhat subtle. The analysis of the leading IC-related contributions in B mesons indicated [22] that the nonperturbative corrections to the vector $\bar{b}b\bar{c}\gamma_0 c$ expectation value dominate, but are partially moderated by those in the axial piece given by $\bar{b}\vec{\sigma}b\bar{c}\vec{\gamma}c$. In Λ_b the axial piece vanishes to the leading order since the light degrees of freedom form a spinless state. The overall IC estimate [22, 23] can readily be adapted to Λ_b by setting $\mu_G^2 = \rho_{LS}^3 = 0$ and using the appropriate values for μ_π^2 , ρ_D^3 and $\bar{\epsilon}_P$. The estimate of the two-loop IC effects $\propto \alpha_s/m_c m_b^3$ can likewise be obtained with the suitable adaptation of the analysis in Ref. [22]. This results in a tentative estimate of a similar effect of the same sign, yet somewhat suppressed by a factor of 0.5 to 0.7. These considerations suggest that

$$\delta_{\text{IC}} \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B)} \gtrsim -0.3\%. \quad (26)$$

A similar analysis can be applied to the regular $1/m_b^4$ and $1/m_b^5$ corrections. We obtain a mild overall suppression of the combined higher-order effect in Λ_b , by about 30% of that in B mesons, thus mirroring the numerics in Eq. (26). This estimate is only tentative, though.

It is interesting to note that the heavy quark expectation values in Λ_b do not appear to be remarkably larger than their B counterparts: the kinetic value emerges close, and the Darwin value is possibly even smaller than in B . This comes in contrast with $\bar{\Lambda}$ which is significantly larger; it evidently scales like N_c in heavy baryons. Through the SV sum rules we conclude that the IW slope must be significantly larger in Λ_b (in particular, once the dynamically-generated slopes are counted on the same footing), the fact almost inevitable in any model. The similarity of the dynamic properties of Λ_b and B in the large- N_c expansion has been emphasized by M. Shifman based on the orientifold approach to the large- N_c QCD [27].

To summarize: Our expectations for $\Gamma_{\text{sl}}(\Lambda_b)/\Gamma_{\text{sl}}(B) - 1$ center around +3% being dominated by the absence of the chromomagnetic suppression in Λ_b and otherwise only slightly affected by the corrections from the Darwin and kinetic operators and from higher-order effects.

3.3 A note on $\Gamma_{\text{sl}}(\Omega_b)$

In a distant future the double-strange Ω_b -baryon may represent an intriguing case for a precision test of the HQE predictions for the decay rates. It is the only stable beauty baryon with a spin-1 light cloud (its hyperfine twin Ω'_b decays electromagnetically), and it is significantly heavier than either B mesons or Λ_b , $M_{\Omega_b} \approx 6.05$ GeV, being a separate light-flavor symmetry state. In spite of the large mass its semileptonic width may not be any noticeably larger than for Λ_b or B – probably, it lies between the two. Based on the measured hyperfine splitting in Ω_c , the width is expected to be suppressed by the

chromomagnetic interaction at the level of 1.5%,

$$\frac{\delta_{\mu_G^2} \Gamma_{\text{sl}}(\Omega_b)}{\Gamma_{\text{sl}}(B_d)} \simeq \frac{8}{9} \frac{M_{\Omega_c^*} - M_{\Omega_c}}{M_{D^*} - M_D} \cdot \frac{\delta_{\mu_G^2} \Gamma_{\text{sl}}(B_d)}{\Gamma_{\text{sl}}(B_d)} \approx -1.5\%, \quad (27)$$

i.e. only about 45% of the hyperfine effect in B_d . At the same time the correction to the width ratio stemming from the kinetic operator must be rather insignificant. The main uncertainty in the ratio $\Gamma_{\text{sl}}(\Omega_b)/\Gamma_{\text{sl}}(B_d)$ is associated with the Darwin expectation value which has not yet been analyzed in detail for the state. Assuming, in the spirit of the most naive nonrelativistic quark models, that the underlying difference with Λ_b is mainly related to the larger s -diquark mass while its spin playing a secondary role, one would expect ρ_D^3 in Ω_b to exceed that in Λ_b . In this case Ω_b may turn out to have the semileptonic width below that of B_d and, potentially, the smallest among all b particles.

The prospects for studying the lifetime of Ω_b at the LHC are more optimistic since the machine should produce a sufficiently large data sets for it. The theoretical predictions for the corrections to the nonleptonic width are not as definite, however, being affected by the significant spectator-specific effects in the KM-allowed modes.

4 Charmless semileptonic decays

So far we have discussed the total widths in the CKM allowed channels $b \rightarrow c e \nu$ and $b \rightarrow c \mu \nu$. The $b \rightarrow c(u) \tau \nu_\tau$ channel and the $b \rightarrow u \ell \nu$ decays are usually considered independently. The effect of the τ mass in $b \rightarrow c \tau \nu_\tau$ does not qualitatively change the predictions, therefore adding it would affect little the overall ratios of the semileptonic widths.

The $b \rightarrow u \ell \nu$ channel is generally suppressed by a factor $2|V_{ub}/V_{cb}|^2 \sim 0.02$; including it in Γ_{sl} would modify the width ratios at the per mil level. Consequently it is more relevant to discuss the ratios of the $b \rightarrow u \ell \nu$ widths directly, for B_d , B_s and Λ_b .

The effect of the kinetic operator is fully universal and does not depend on the channel at all. The chromomagnetic coefficient depends, strictly speaking, on the quark mass in the final state, but this dependence is not spectacular: the charm mass enhances the chromomagnetic effect compared to the $b \rightarrow u$ case by a factor about 1.35. Therefore, one obtains here the same small effect as for the $b \rightarrow c \ell \nu$ widths.

The largest splitting comes from the Darwin operator which, for $b \rightarrow u \ell \nu$ is entangled with the $SU(3)$ -breaking between B_d and B_s in the non-valence WA. In Λ_b the latter corresponds to what is conventionally referred to as the PI mechanism. These actually comprise the counterpart of the IC corrections discussed for $b \rightarrow c \ell \nu$; in the higher-order corrections only the ‘regular’ terms suppressed by powers of $1/m_b$ should be considered.

A recent discussion of the valence and non-valence WA effects in B mesons can be found in Ref. [23]. Both were estimated as negative yet quite small, around -1.5% . Even though tentative, this suggests that the actual effect in the $SU(3)$ breaking between B_s and B_d will be dominated by the difference in the Darwin expectation value,

$$\delta \frac{\Gamma_{\text{sl}}(B_s \rightarrow X_u \ell \nu)}{\Gamma_{\text{sl}}(B_d \rightarrow X_u \ell \nu)} \approx -(1.5 \text{ to } 3)\%, \quad (28)$$

with a totally negligible impact on the combined ($b \rightarrow c \ell \nu$ and $b \rightarrow u \ell \nu$) semileptonic width.

Turning to $\Gamma_{\text{sl}}(\Lambda_b \rightarrow X_u \ell \nu)$ the previous reasoning is amended by the potentially largest effect, the direct PI in Λ_b

$$\delta_{\text{PI}} \frac{\Gamma_{\text{sl}}(\Lambda_b \rightarrow X_u \ell \nu)}{\Gamma_{\text{sl}}(B_d \rightarrow X_u \ell \nu)} \simeq \frac{32\pi^2}{m_b^3} \frac{1}{2M_{\Lambda_b}} \langle \Lambda_b | \bar{b} \vec{\gamma} (1 - \gamma_5) u \bar{u} \vec{\gamma} (1 - \gamma_5) b | \Lambda_b \rangle \quad (29)$$

which – in contrast to WA in B – is not prone to chirality suppression ab initio. Neglecting the non-valence s -quark effect (apparently suppressed) Ref. [26] estimated the above expectation value to be about $2\lambda' - \frac{1}{2}\lambda = 0.025 \text{ GeV}^3$. Adopting this we obtain

$$\delta_{\text{PI}} \frac{\Gamma_{\text{sl}}(\Lambda_b \rightarrow X_u \ell \nu)}{\Gamma_{\text{sl}}(B_d \rightarrow X_u \ell \nu)} \approx 8.5\%, \quad (30)$$

notably the dominant enhancement effect. The $b \rightarrow u$ semileptonic width difference with B_d is then expected about 10%:

$$\frac{\Gamma_{\text{sl}}(\Lambda_b \rightarrow X_u \ell \nu)}{\Gamma_{\text{sl}}(B_d \rightarrow X_u \ell \nu)} \simeq 1.1; \quad (31)$$

this still may only slightly offset, by 0.2% our estimate of the combined semileptonic width of Λ_b as compared to B_d .

5 The semileptonic widths in the phase-space model

The paper [1] let us look again into the numerics of the QCD-based HQE predictions for the splitting between the b -particle semileptonic decay widths. The paper used a simple model for hadronization corrections derived solely from the phase-space effects in the decays into the lowest pseudoscalar and vector final state mesons (or into the ground-state baryon, for the Λ_b decays), which gave

$$\frac{\Gamma_{\text{sl}}(B_s)}{\Gamma_{\text{sl}}(B_d)} = 1.03, \quad \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B_d)} = 1.13. \quad (32)$$

Not surprisingly, these differ from what the HQE predicts: it has been known from the early 1990s that the phase-space-based calculations generally yield power corrections scaling like $1/m_Q$ and these depend on $\bar{\Lambda}$, the energy gap distinguishing the hadron mass from the heavy quark one. This parameter is the principal nonperturbative quantity in differentiating the beauty hadrons in question. It is instructive to examine the model [1] from this perspective; this allows to readily understand the numeric pattern in Eq. (32).

The $b \rightarrow c \ell \nu$ -mediated decays is the most transparent case where the expansion in both $1/m_b$ and in $1/m_c$ applies. One only needs to employ the hadron mass expansion

$$\begin{aligned} M_B &= m_b + \bar{\Lambda} + \frac{\mu_\pi^2 - \mu_G^2}{2m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right) + \dots, & M_{B^*} &= m_b + \bar{\Lambda} + \frac{\mu_\pi^2 + \frac{1}{3}\mu_G^2}{2m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right) + \dots, \\ M_{\Lambda_b} &= m_b + \bar{\Lambda}_\Lambda + \frac{\mu_\pi^2(\Lambda_b)}{2m_b} + \mathcal{O}\left(\frac{1}{m_b^2}\right) + \dots \end{aligned} \quad (33)$$

and likewise for charm. The corrections to the decay width assuming the absence of the formfactors and the 3:1 ratio of the D^* to D yields (the adopted assumptions in Ref. [1]) then take the form

$$\Gamma_{\text{sl}}(H_b) \simeq \Gamma_{\text{sl}}(b) \left[1 + C_{\bar{\Lambda}} \frac{\bar{\Lambda}}{m_b} + C_{\pi} \frac{\mu_{\pi}^2}{2m_b^2} + C_G \frac{\mu_G^2}{2m_b^2} + \dots \right], \quad (34)$$

a series extended to any desired level. In particular, there is a strong dependence on $\bar{\Lambda}$, $C_{\bar{\Lambda}} \simeq 1.77$ that would dominate the width differences. The total width in such a model as a function of $\bar{\Lambda}$ is well approximated by this linear dependence.

Using $\bar{\Lambda}_{\Lambda} - \bar{\Lambda} \simeq M_{\Lambda_b} - \bar{M}_B \simeq 300 \text{ MeV}$ Eq. (34) would yield

$$\delta_{\bar{\Lambda}} \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B_d)} \simeq 11.5\%; \quad (35)$$

i.e., it reproduces the bulk of the Ref. [1] prediction of 13%.

At first glance, there is something strange with the smaller difference for B_s where scaling the above estimate by the ratio of $\delta\bar{\Lambda}$ one would estimate

$$\delta \frac{\Gamma_{\text{sl}}(B_s)}{\Gamma_{\text{sl}}(B_d)} \approx \frac{M_{B_s} - M_B}{M_{\Lambda_b} - M_B} \cdot \delta \frac{\Gamma_{\text{sl}}(\Lambda_b)}{\Gamma_{\text{sl}}(B_d)} \approx \frac{87 \text{ MeV}}{339 \text{ MeV}} \cdot 0.13 \approx 3\% \quad (36)$$

vs. 1.2% obtained in Ref. [1]. A closer look reveals the origin of the reduction from 3% to 1.2%: it is an effect of the kinetic term in Eq. (34). In the phase-space approximations the coefficient C_{π} typically is very large. Here at $m_c = 1.25 \text{ GeV}$ and $m_b = 4.6 \text{ GeV}$ it comes out

$$C_{\pi} \approx -15; \quad (37)$$

on the contrary, the OPE ensures the universal value of $c_{\pi} = -1$ regardless of the underlying dynamics. Using the value in Eq. (9) for the μ_{π}^2 splitting – that would stem from the literal comparison of the meson masses – the inflated C_{π} value in Eq. (37) yields a -1.5% correction to be added to the $\bar{\Lambda}$ effect in Eq. (36). (We can note that for some partially accidental reason the effect of the chromomagnetic term C_G comes out approximately correct for the semileptonic width in the case of $V-A$ weak interaction [4, 5] provided the D^* to D yield is taken 3:1 canceling the phase space effect in the final state.)

Thus we conclude that the moderate size of the correction in $\Gamma_{\text{sl}}(B_s)$ in the phase-space model comes as a result of a cancellation between the $\bar{\Lambda}/m_b$ term and a 15-fold inflated effect of the kinetic operator. Neither are in the widths in reality.

The above heavy mass expansion has certain peculiarities in the case of $b \rightarrow u \ell \nu$ widths. Here Eq. (34) takes the form

$$\Gamma_{\text{sl}}(H_b \rightarrow X_u \ell \nu) \simeq \Gamma_{\text{sl}}(b \rightarrow u \ell \nu) \cdot \left[1 + 5 \frac{\bar{\Lambda}}{m_b} - 8 \frac{\sum w_i M_i^2}{m_b^2} + \left(10 \frac{\bar{\Lambda}^2}{m_b^2} + \frac{5\mu_{\pi}^2}{2m_b^2} \right) + \dots \right] \quad (38)$$

where M_i are the meson masses in the final state and w_i the corresponding branching fractions. In QCD the equivalent of the sum $\sum w_i M_i^2$ equals to $\frac{5}{8} \bar{\Lambda} m_b$; the exact factor

actually depends on the kinematics precisely in such a way that the second and the third term cancel at arbitrary q^2 of the lepton pair. However, the phase-space model of Ref. [1] considers only the lowest pseudoscalar and vector mesons with light quarks; then the third term inevitably scales like $1/m_b^2$. As a result, the $5\bar{\Lambda}/m_b$ correction remains unabated, at least for sufficiently heavy decaying hadron.

It is nevertheless interesting to note that the model intrinsically includes an effect of WA in mesons estimated to be about -2% for the $b \rightarrow u \ell \nu$ rates. No mechanism for the annihilation proper was actually introduced, and the whole effect belongs to the realm of the ‘generalized WA’ phenomenon put forward in Ref. [2], related, in particular, to the annihilation-driven upward shift in the pseudoscalar masses. This readily explains the negative sign of the ‘annihilation’ correction to the width. The thus generated WA is rooted in the QCD’s $U(1)$ -problem and is expressed through the anomalous mass square of the η' meson.

At the same time, the WA in the model of Ref. [1] effectively scales like $1/m_b^2$ whereas in reality it must be $1/m_b^3$. The reason is that all the $b \rightarrow u$ width in the model is attributed to the lowest π and ρ states, together with their unitary siblings. On the contrary, in QCD these individual states are responsible for only a $1/m_b^3$ fraction of the decay probability. Another distinct kinematic feature is that the related flavor-dependent corrections are present for all kinematics, including fast π_0, η, η' with $|\vec{p}| \approx \frac{m_b}{2}$. The leading contribution to WA in QCD originates from the domain of low-momentum hadronic final states.

Finally, let us note that another WA-like mechanism – the PI in the $\Lambda_b \rightarrow X_u \ell \nu$ decays – appears to be the largest preasymptotic correction in beauty, unrelated to any mass shift. No room for such effect is seen in the phase-space models.

5.1 On the $\bar{\Lambda}$ effects

The main fact of the HQE in QCD is the absence of the $\bar{\Lambda}/m_b$ corrections in the inclusive widths, which underlies the principal numerical difference with the phenomenological models that emphasize the phase-space effects. In particular, the latter are enhanced by the large power of the heavy quark mass commanding the partonic width, that naively does not show up in the transition amplitudes. This aspect of the QCD result was elucidated in Ref. [28].

Comparing the actual QCD with the phase-space model, notably for the $b \rightarrow c \ell \nu$ transitions where both b and c quarks are treated as heavy, one observes that the hadronic transition amplitudes into D and D^* are additionally suppressed nonperturbatively, at nonvanishing recoil, by the corresponding formfactors. These effects at a given velocity do not fade away even for infinitely heavy quarks. This would have led to the mismatch between the partonic and hadronic calculations already at a $\mathcal{O}(1)$ level. As noted in the mid 1980s by Voloshin and Shifman [29], this is eliminated by the production of the truly excited charmed states. In the SV regime these are the various P -wave states, and the cancellations of the $1/m_Q^0$ corrections in the SV limit is expressed by the Bjorken sum

rule [30]

$$\varrho^2 - \frac{1}{4} = \sum_n 2|\tau_{3/2}^{(n)}|^2 + \sum_n |\tau_{1/2}^{(n)}|^2. \quad (39)$$

P -waves are not included in the phase-space model of Ref. [1], yet neither are the formfactor effects which should amount to setting the IW slope $\varrho^2 = \frac{1}{4}$ in B (or to zero in Λ_b). The precise shapes assumed for the formfactors are actually unimportant in the calculations of the widths ratios at this leading level, only that they are the same.

That becomes less obvious for the terms $\overline{\Lambda}/m_b$. It was detailed in Ref. [31] that the absence of such corrections in the widths comes from the sum rule for the first moment of the structure functions; in the SV regime this becomes the ‘optical’ sum rule from Voloshin [32]:

$$\frac{\overline{\Lambda}}{2} = \sum_n 2\varepsilon_n |\tau_{3/2}^{(n)}|^2 + \sum_n \varepsilon_n |\tau_{1/2}^{(n)}|^2 \quad (40)$$

including, again, only the final states with the P -wave quantum numbers. The relation, however is more general and ensures vanishing of the $\overline{\Lambda}/m_b$ effects for the arbitrary final state quark mass.

There have been later analyses focusing on the way the cancellation occurs upon combining the ground-state and the P -wave state yields in B -decays in the SV regime. In particular, Ref. [33] scrutinized a more complicated case of the axial-vector transitions.⁴ A dedicated review presentation can be found in Ref. [34].

Now the inherent problem of the phase-space models should become evident: lacking a dynamic description of the transition formfactors and of the P -wave amplitudes they have to assume the structureless ground-state formfactors and absent P -wave transitions, to save duality to the leading order in $1/m_Q$. However, this only pushes the problem to the next order $1/m_Q$. Once the P -wave amplitudes are absent, there may be no difference between the quark and hadron kinematics: $\overline{\Lambda}$ has to vanish. Adopting the absence of the P -wave transitions and non-zero $\overline{\Lambda}$ simultaneously can not be reconciled in any quantum mechanical system.

6 Conclusions

We have presented our expectations for the ratios of the semileptonic decay rates of B_d , B_s and Λ_b hadrons applying the OPE-based $1/m_Q$ expansion. In $\Gamma_{\text{sl}}(B_s)/\Gamma_{\text{sl}}(B_d)$ the effect originates from the $SU(3)$ breaking in the heavy quark expectation values. While generally the $SU(3)$ splittings may be expected to constitute 30 to 50%, we found a peculiar pattern: the leading terms in the expansion appear to be far more $SU(3)$ -robust, especially those describing the spin effects. The full-strength $SU(3)$ breaking apparently is delayed to higher orders in $1/m_Q$, thereby suppressing the overall $SU(3)$ violating effects in the beauty mesons.

⁴A. Vainshtein had presented a similar sum rule, but never published it.

We expect a slower semileptonic decay of B_s mesons compared to B_d , by about 1%, mainly due to a larger Darwin expectation value $\rho_D^3(B_s)$. Other effects derived from the observed heavy meson masses appear to be notably smaller.

The prediction $\delta\Gamma_{\text{sl}}(B_s)/\Gamma_{\text{sl}}(B_d) \approx -1\%$ is tied to the expected larger value for the Darwin operator in B_s . This leads to other implications, for instance a larger slope of the IW formfactor for the strange meson, and calls for the independent verifications.

This finding is peculiar from a certain perspective: the natural scale for the $SU(3)$ breaking in the individual exclusive channels is generally described by a parameter like $f_K^2/f_\pi^2 \approx 1.4$; this is much larger than what is seen in the inclusive rates. Therefore, the duality with the OPE description assumes nontrivial cancellations between the various exclusive channels to ensure only a small $SU(3)$ breaking in their sum.

No symmetry promotes equality of the decay rates in Λ_b and B . Nevertheless the OPE predicts only a small difference. The largest effect, about 3% is the absence in Λ_b of the suppression from the chromomagnetic interaction. A potentially significant contribution to the width difference from the Darwin operator appears suppressed according to the dynamics based estimated.

In $b \rightarrow u \ell \nu$ semileptonic widths the deviation from unity of the B_s to B_d ratio is expected to be small, $\delta\Gamma(B_s \rightarrow X_u \ell \nu)/\Gamma(B_d \rightarrow X_u \ell \nu) \approx -2.5\%$, although this number reflects somewhat model-dependent estimates [23] of the non-valence WA. At the same time a similar effect – the PI in Λ_b in the $b \rightarrow u \ell \nu$ channel – emerges as the largest effect driving up the ratio to

$$\frac{\Gamma(\Lambda_b \rightarrow X_u \ell \nu)}{\Gamma(B_d \rightarrow X_u \ell \nu)} \approx 1.1.$$

In an even more remote perspective, we have found semileptonic Ω_b decays to be of a particular interest. Its semileptonic width is expected to be slightly suppressed and close to that of B_d meson; it may even turn out the smallest among all b hadrons – in spite of the remarkably large mass of Ω_b . The uncertainty in the OPE prediction for $\Gamma_{\text{sl}}(\Omega_b)/\Gamma_{\text{sl}}(B_d)$ may be reduced once their masses along with the hyperfine partner about 23 MeV higher are accurately measured, and if the information on their P -wave states in the beauty or charm sectors becomes available.

The OPE-based predictions appear quite different from the phase-space models. For B_s we expect a smaller effect of the opposite overall sign; in $\Gamma_{\text{sl}}(\Lambda_b)$ the phase-space model cannot avoid a much larger effect of about 13% due to the term scaling like $\bar{\Lambda}/m_b$. In the $b \rightarrow u \ell \nu$ channel the significant effect, up to 10% comes in the OPE from the flavor-dependent processes involving the interference with the spectator quark, a mechanism having no counterpart in the phase-space picture.

The ideas to reduce the bulk of the hadronization effects in the inclusive heavy flavor decays to the phase-space corrections alone have a long history. Perhaps the best known was paper [36] which sought to explain the existed significant experimental difference in the Λ_b vs. B_d lifetimes by overriding the OPE results with the assumed $M_{H_Q}^5$ scaling of the widths. The evident fact that baryons in the decay products of Λ_b have likewise larger mass than their meson counterparts, was discarded. This particular drawback has been eliminated in Ref. [1]. Accounting for the increase in the final-state mass reduced, as

expected, the shift in the resulting (semileptonic) decay rate ratio $\Gamma_{\text{sl}}(\Lambda_b)/\Gamma_{\text{sl}}(B_d)$ from 1.37 to 1.13, a value much closer to the OPE expectation – yet still markedly above it.

Acknowledgments

We thank A. Khodjamirian for useful discussions and S. Turczyk for help with evaluating the estimated higher-order power effects. The communication from M. Artuso raising the problem in question is acknowledged. The study enjoyed a partial support from the NSF grant PHY-0807959 and from the RSGSS-65751.2010.2 grant. TM is partially supported by the German research foundation DFG under contract MA1187/10-1 and by the German Ministry of Research (BMBF), contracts 05H09PSF.

References

- [1] M. Gronau and J.L. Rosner, arXiv:1012.5098 [hep-ph].
- [2] I.I. Bigi and N.G. Uraltsev, Nucl. Phys. **B423** (1994) 33 and Z. Phys. C **62** (1994) 623.
- [3] N. Uraltsev, Int. J. Mod. Phys. A **14** (1999) 4641.
- [4] I.I. Bigi, N.G. Uraltsev and A.I. Vainshtein, Phys. Lett. B **293** (1992) 430.
- [5] B. Blok and M.A. Shifman, Nucl. Phys. B **399** (1993) 441.
- [6] N.N. Nikolaev, Pisma Zh. Eksp. Teor. Fiz. **18**, 447 (1973).
- [7] By far incomplete sample of references includes
H. Fritzsch and P. Minkowski, Phys. Lett. B **90** (1980) 455;
W. Bernreuther, O. Nachtmann and B. Stech, Z. Phys. C **4** (1980) 257;
B. Guberina, R. D. Peccei and R. Ruckl, Phys. Lett. B **91** (1980) 116;
T. Kobayashi and N. Yamazaki, Prog. Theor. Phys. **65** (1981) p.775;
G. Altarelli and L. Maiani, Phys. Lett. B **118** (1982) 414;
N. Bilic, B. Guberina and J. Trampetic, Nucl. Phys. B **248** (1984) 261;
M.A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. **41**, 120 (1985) [Yad. Fiz. **41**, 187 (1985)]; Sov. Phys. JETP **64**, 698 (1986) [Zh. Eksp. Teor. Fiz. **91**, 1180 (1986)].
- [8] I.I. Bigi and N.G. Uraltsev, Phys. Lett. B **280** (1992) 271.
- [9] D. Benson et al., Nucl. Phys. B **665** (2003) 367.
- [10] L. Koyrakh, Phys. Rev. D **49** (1994) p.3379-3384.
- [11] T. Mannel, S. Turczyk and N. Uraltsev, *JHEP* **1007** (2010) 109.
- [12] I.I. Bigi et al., Phys. Rev. Lett. **71** (1993) 496.

- [13] R.F. Lebed and N.G. Uraltsev, Phys. Rev. D **62** 094011 (2000) 094011.
- [14] G. 't Hooft, Nucl. Phys. B **75** (1974) 461.
- [15] N. Uraltsev, Phys. Lett. B **545** (2002) 337.
- [16] P. Gambino, T. Mannel and N. Uraltsev, *Phys. Rev.* **D81** 113002 (2010); P. Gambino et al., *in writing*.
- [17] I.I. Bigi et al., Int. J. Mod. Phys. A **9** (1994) 2467.
- [18] I. Bigi et al., Phys. Rev. D **59** (1999) 054011.
- [19] M. Burkardt and N. Uraltsev, Phys. Rev. D **63** (2001) 014004.
- [20] A.E. Blinov, N.G. Uraltsev and V.A. Khoze, Sov. Phys. JETP **70**, 32 (1990) [Zh. Eksp. Teor. Fiz. **97**, 59 (1990)]; Preprint LNPI-89-0023, Leningrad, 59pp;
V.A. Khoze and N.G. Uraltsev, Sov. J. Nucl. Phys. **47**, 1069 (1988) [Yad. Fiz. **47**, 1687 (1988)].
- [21] M. Gremm and A. Kapustin, Phys. Rev. D **55** (1997) p.6924-6932.
- [22] I.I. Bigi, N. Uraltsev and R. Zwicky, Eur. Phys. J. C **50** (2007) 539.
- [23] I. Bigi et al., *JHEP* **1004** (2010) 073.
- [24] N. Uraltsev, Phys. Lett. B **501** (2001) 86.
- [25] N. Uraltsev, Phys. Lett. B **585** (2004) 253.
- [26] D. Pirjol and N. Uraltsev, Phys. Rev. D **59** (1999) 034012.
- [27] A. Armoni, M. Shifman and G. Veneziano, Nucl. Phys. B **667** (2003) 170.
- [28] I.I. Bigi et al., Phys. Rev. D **56** (1997) 4017.
- [29] M.A. Shifman and M.B. Voloshin, Sov. J. Nucl. Phys. **47**, 511 (1988) [Yad. Fiz. **47**, 801 (1988)].
- [30] J. D. Bjorken, in: *Results and perspectives in particle physics*, Proceedings of the 4th Rencontre de la Valle d'Aoste, La Thuile, Italy, Mar 18-24, 1990 ed. M. Greco (Editions Frontières, Gif-sur-Yvette, France 1990) p. 583.
- [31] I.I. Bigi et al., Phys. Rev. D **51** (1995) 2217.
- [32] M.B. Voloshin, Phys. Rev. D **46** (1992) 3062.
- [33] A. Le Yaouanc et al., Phys. Lett. B **480** (2000) 119.
- [34] I.I. Bigi and T. Mannel, arXiv:hep-ph/0212021 (Contributed to Ref. [35], Sect. 3.2.3).

- [35] M. Battaglia *et al.*, arXiv:hep-ph/0304132. Proceedings of the CKM Workshop, Edited by M. Battaglia, A.J. Buras, P. Gambino, A. Stocchi, Geneva, CERN, 2003, 271pp.
- [36] G. Altarelli et al., Phys. Lett. B **382** (1996) 409.